Separation Science and Technology
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713708471

Optimum Plate-Spacing for the Best Performance of the Enrichment of Heavy Water in Flat-Plate Thermal-Diffusion Columns of the Frazier Scheme

Ho-Ming Yeh[a]
[a] DEPARTMENT OF CHEMICAL ENGINEERING, TAMKANG UNIVERSITY, TAMSUI, TAIWAN, REPUBLIC OF CHINA

To cite this Article Yeh, Ho-Ming(1996) 'Optimum Plate-Spacing for the Best Performance of the Enrichment of Heavy Water in Flat-Plate Thermal-Diffusion Columns of the Frazier Scheme', Separation Science and Technology, 31: 18, 2543 — 2556
To link to this Article: DOI: 10.1080/01496399608001064
URL: http://dx.doi.org/10.1080/01496399608001064

SEPARATION SCIENCE AND TECHNOLOGY, 31(18), pp. 2543–2556, 1996

Optimum Plate-Spacing for the Best Performance of the Enrichment of Heavy Water in Flat-Plate Thermal-Diffusion Columns of the Frazier Scheme

HO-MING YEH
DEPARTMENT OF CHEMICAL ENGINEERING
TAMKANG UNIVERSITY
TAMSUI, TAIWAN, REPUBLIC OF CHINA

ABSTRACT

The effect of plate spacing on the degree of separation and production rate for the enrichment of heavy water in flat-plate thermal diffusion columns of the Frazier scheme with fixed operating expense has been investigated. The equations for estimating optimum plate-spacing for maximum separation and for maximum production rate have been developed. Considerable improvement in performance is obtainable when thermal diffusion columns with optimum plate-spacing are employed for operation.

INTRODUCTION

Heavy water (D_2O) is the mosst feasible moderator and coolant for fission reactors to furnish excess neutrons which may be absorbed in materials other than uranium, and may be the indirect nuclear fuel for fusion reactors. Between 1940 and 1945, four heavy-water production plants were placed in operation by the US Government under the Manhattan District Program (1, 6).

Thermal diffusion is a well-established methos for separating isotopes, especially for isotope mixtures with a large ratio in molecular weight. During 1938 and 1939, Clusius and Dickel pointed out that heavy water can be concentrated successfully in thermal diffusion columns (2, 3). The enrichment of heavy water in thermal diffusion columns was studied both theoretically and experimentally by the present author (9–12). More recently, it was also found that considerable improvement in separation of the H_2O–HDO–D_2O system could be achieved in a vapor-phase thermal diffusion column (13).

In industrial applications, thermal diffusion columns are connected in series, such as that shown in Fig. 1, called the Frazier scheme (4). The separation theory for the enrichment of heavy water in inclined thermal diffusion columns of the Frazier scheme has been developed recently in previous papers (15, 16). The equations for the best angle of inclination and maximum separation have been derived. Considerable improvement in separation is obtainable if the columns are inclined at the best angle,

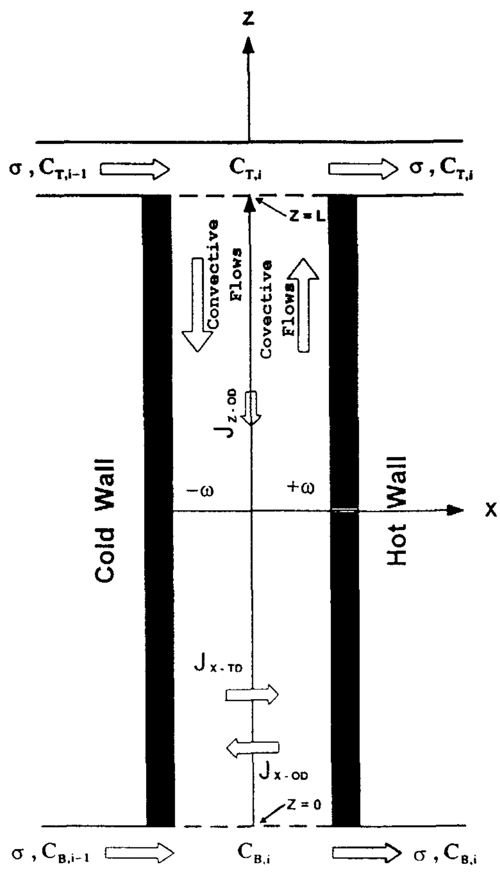

FIG. 1 Schematic diagram of the Frazier scheme for forward transverse flow.

so that the convective strength can be properly reduced and controlled, resulting in suppression of the undesirable remixing effect while still preserving the desirable cascading effect.

Although a number of studies of operating variables in thermal diffusion columns have been made, plate spacing, which has an important effect and apparently affects separation efficiency, is hardly ever discussed. Recently, the optimum plate-spacing for the best performance in conventional Clusius-Dickel columns has been investigated (14). It is the purpose of this work to investigate the effect of plate spacing on the degree of separation and production rate for the enrichment of heavy water in thermal diffusion columns of the Frazier scheme under consideration of a fixed operating expense.

SEPARATION EQUATIONS

The scheme proposed by Frazier to connect several vertical thermodiffusion columns of the same size with forward transverse sampling streams is shown in Fig. 1. The delivery of supply σ with concentration C_0 is accomplished at the upper and lower ends in a plane thermodiffusion column with gap 2ω, length L, and width B, where both streams move in the same direction. Sampling of the product is carried out at the ends opposite to the supply entrance. Frazier (4) gave a theory of this process on the basis of a simplified representation, while an analytical solution was given by Rabinovich and Suvorov (7, 8).

The temperature gradient applied between the surfaces of a thermogravitational thermal diffusion column has two effects: 1) a flux of one component of the solution relative to the other is brought about by thermal diffusion, and 2) natural convective currents are produced parallel to the plates owing to density differences. The combined result of these effects is to produce a concentration difference between the two ends of the column. Meanwhile, the concentration gradient produced by the combined effects of thermal diffusion and convection acts to oppose thermal diffusion and to limit the separation. Figure 2 illustrates the flows and fluxes prevailing in the ith thermal diffusion column of the Frazier scheme. The separation theory of thermal diffusion was first presented by Furry et al. (5). They derived the transport equation for the component which is concentrated toward the top of a thermogravitational thermal diffusion column (say the ith column) as

$$\tau_i = HC(1 - C) - K\frac{dC_i}{dz} > 0 \tag{1}$$

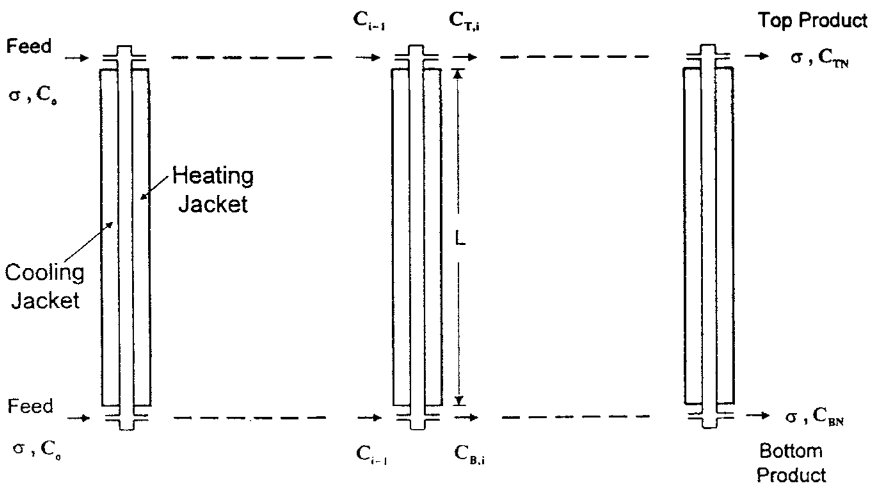

FIG. 2 Flows and fluxes previling in the *i*th thermal diffusion column of the Frazier scheme.

where

$$H = \frac{\alpha\rho\beta g(2\omega)^3 B(\Delta T)^2}{6!\mu T_m} \tag{2}$$

$$K = \frac{\rho\beta^2 g^2(2\omega)^7 B(\Delta T)}{9!D\mu^2} \tag{3}$$

In obtaining the above equations, since the degree of separation obtained in a thermal diffusion column is generally small, the product of the concentration terms, $C(1 - C)$, was treated as a constant, say, $C_0(1 - C_0)$. For the enrichment of heavy water by thermal diffusion in the H_2O–HDO–D_2O system coupled with the chemical equilibrium $H_2O + D_2O \rightarrow 2HDO$, D_2O is concentrated toward the bottom rather than toward the top of the column, i.e., $\alpha < 0$ (or $H < 0$). Further, Yeh and Yang (9) assumed that the concentrations were locally in equilibrium at every point in thermal diffusion columns, and thus the transport equation for heavy water was obtained as

$$\tau_i = HA - K\frac{dC_i}{dz} < 0 \tag{4}$$

where

$$A = C_0[0.05263 - (0.05263 - 0.0135K_{eq})C_0 - 0.027\{C_0K_{eq}(1 - (1 - 0.25K_{eq})C_0)\}^{1/2}] \tag{5}$$

in which K_{eq} is the mass fractional equilibrium constant of the H_2O–HDO–D_2O system.

By making material balances for the top and the bottom of the column, one obtains, respectively,

$$-\tau_i = -HA + K\left.\frac{dC_i}{dz}\right|_{z=L} = \sigma(C_{T,i-1} - C_{T,i}) \tag{6}$$

$$-\tau_i = -HA + K\left.\frac{dC_i}{dz}\right|_{z=0} = \sigma(C_{B,i} - C_{B,i-1}) \tag{7}$$

Further, if a material balance is taken for the whole column, the result is

$$C_{T,i-1} - C_{T,i} = C_{B,i} - C_{B,i-1} \tag{8}$$

Combination of Eqs. (6)–(8) results in

$$\left.\frac{dC_i}{dz}\right|_{z=L} = \left.\frac{dC_i}{dz}\right|_{z=0} \tag{9}$$

Thus, we may assume that

$$dC_i/dz = \text{constant} \tag{10}$$

within the whole column. By integrating Eqs. (6) and (7) from the top to the bottom of the column, the following two equations are obtained:

$$C_{T,i} - C_{B,i} = \left[A - \frac{\sigma}{H}(C_{T,i-1} - C_{T,i})\right]\frac{HL}{K} \tag{11}$$

$$C_{T,i} - C_{B,i} = \left[A - \frac{\sigma}{H}(C_{B,i} - C_{B,i-1})\right]\frac{HL}{K} \tag{12}$$

Addition of Eqs. (11) and (12) gives the equation for calculating the degree of separation obtainable in the ith column as

$$\Delta_i - \left[\left(\frac{\sigma L}{2K}\right)^{-1} - 1\right]^{-1}\Delta_{i-1} - \frac{AHL}{K}\left[\left(\frac{\sigma L}{2K}\right) - 1\right]^{-1} = 0 \tag{13}$$

where

$$\Delta_i = C_{B,i} - C_{T,i} \tag{14}$$

Equation (13) is the first-order difference equation of Δ_i whose solution, subject to the inlet and outlet conditions:

$$\left.\begin{array}{ll} i = 0, & C_{T,0} = C_{B,0} = C_0 \\ i = N, & C_{B,i} = C_{B,N},\ C_{T,i} = C_{T,N} \end{array}\right\} \tag{15}$$

is

$$\Delta_N = C_{B,N} - C_{T,N}$$
$$= \frac{A(-H)L}{K}\left\{1 - \left[1 + \left(\frac{2K}{\sigma L}\right)\right]^{-N}\right\} \tag{16}$$

in which Δ_N denotes the degree of separation obtainable in a Frazier scheme with N thermal-diffusion columns.

In obtaining the above solution, assumptions were made that all the perimeters of the columns were the same in magnitude and that the rates of the sampling streams were all the same as σ.

THE EFFECT OF PLATE SPACING ON PERFORMANCE

The plate spacing (2ω) in a thermal diffusion column is generally so small that changing 2ω will not cause any additional fixed charge. The expenditure of making a separation by thermal diffusion essentially includes two parts: a fixed charge and an operating expense. The fixed charge is roughly proportional to the equipment cost, while the operating expense is chiefly heat. The transfer rate is obtainable from the expression, $KBL(\Delta T/2\omega)$. Based on these terms, we shall account of the influence of plate-spacing change on the degree of separation and production rate with consideration of the fixed operating expense.

Accordingly, with consideration of the fixed operating expense ($\Delta T/2\omega$ = constant), Eqs. (2) and (3) may be rewritten as

$$a = \frac{\alpha\rho\beta g B(\Delta T/2\omega)^2}{6!\mu T_m} = H/(2\omega)^5 = \text{constant} \tag{17}$$

$$b = \frac{\rho\beta^2 g^2 B(\Delta T/2\omega)^2}{9!D\mu^2} = K/(2\omega)^9 = \text{constant} \tag{18}$$

and Eq. (16) becomes

$$\Delta_N = \left[\left(\frac{-aLA}{b}\right)\Big/(2\omega)^4\right]\{1 - [1 + (2b/\sigma L)(2\omega)^9]^{-N}\} \tag{19}$$

Maximum Separation

The optimum plate spacing $(2\omega)^*_1$ for maximum separation $\Delta_{N,max}$ is obtained by partially differentiating Eq. (19) with respect to 2ω and setting $\partial\Delta_N/\partial(2\omega) = 0$. After differentiation and simplification, one obtains

$$4W_\Delta^{N+1} - (9N + 4)W_\Delta + 9N = 0 \tag{20}$$

where

$$W_\Delta = 1 + (2b/\sigma L)(2\omega)_\Delta^{*9} \tag{21}$$

Therefore, the optimum plate spacing for maximum separation can be calculated from

$$(2\omega)_\Delta^* = [(\sigma L/2b)(W_\Delta - 1)]^{1/9} \tag{22}$$

and the expression for maximum separation is obtained by substituting Eq. (21) or (22) into Eq. (19):

$$\Delta_{N,\max} = \left(\frac{-aLA}{b}\right)\left[\left(\frac{\sigma L}{2b}\right)(W_\Delta - 1)\right]^{-4/9}(1 - W_\Delta^{-N}) \tag{23}$$

Note that the value of W_Δ in the above equations is determined from Eq. (20) once the number of columns N employed is decided.

Maximum Production Rate

To find the maximum production rate $\sigma_{\max}$ obtainable to accomplish a specified degree of separation Δ_N, column number N, and column length L, we rearrange Eq. (19) into a form explicit in the production rate

$$\sigma = \frac{(2b/L)(2\omega)^9}{\left[1 - \left(\frac{b}{-aLA}\right)\Delta_N(2\omega)^4\right]^{-1/N} - 1} \tag{24}$$

Maximization of σ with respect to 2ω at constant Δ_N and L yields

$$9W_\sigma^{(N+1)/N} - \left(9 + \frac{4}{N}\right)W_\sigma + \frac{4}{N} = 0 \tag{25}$$

where

$$W_\sigma = 1 - \left(\frac{b}{-aLA}\right)\Delta_N(2w)_\sigma^{*4} \tag{26}$$

Therefore, the optimum plate spacing for the maximum production rate can be calculated from

$$(2\omega)_\sigma^* = \left[\left(\frac{-aLA}{b\Delta_N}\right)(1 - W_\sigma)\right]^{1/4} \tag{27}$$

and the expression for the maximum separation is obtained by substituting

Eq. (26) or (27) into Eq. (24):

$$\sigma_{max} = \frac{(2b/L)[(-aLA/b\Delta_N)(1 - W_\sigma)]^{9/4}}{W_\sigma^{-1/N}} \tag{28}$$

Note again that the value of W_σ in the above equations is determined from Eq. (25) once the number of columns N employed is decided.

Numerical Example

For the purpose of illustration, let us employ some numerical values obtained in previous work (9) for the enrichment of D_2O in the H_2O–HDO–D_2O system as follows: $\Delta T = 47 - 14 = 33°C$; $T_m = 30.5°C$; $K_{eq} = 3.793$; $2\omega = 0.016$ in. $= 0.0406$ cm; $\Delta T/2\omega = 812.8°C/cm$ (fixed operating expense); $L = 177$ cm; $B = 10$ cm; $H = -1.473 \times 10^{-4}$ g/s $= -0.53$ g/s; $K = 1.549 \times 10^{-3}$ g-cm/s $= 5.576$ g-cm/h.

Accordingly, from Eqs. (5), (17), and (18):

$$A \times 10^2 = 0.359\ (C_0 = 0.1),\ 0.709\ (C_0 = 0.3),\ 0.761\ (C_0 = 0.5),\ 0.591\ (C_0 = 0.7),\ 0.237\ (C_0 = 0.9) \tag{29}$$

$$-a = \frac{0.53}{(0.0406)^5} = 4.8 \times 10^6 \text{ g/(h)(cm)}^5 \tag{30}$$

$$b = \frac{5.576}{(0.0406)^9} = 1.86 \times 10^{13} \text{ g/(h)(cm)}^{12} \tag{31}$$

From these values the maximum separation $\Delta_{N,max}$, the maximum production rate σ_{max}, and their best corresponding plate spacing, $(2\omega)^*_\Delta$ and $(2\omega)^*_\sigma$, are calculated from the appropriate equations. Further, the differences of plate temperature, $(\Delta T)_\Delta$ and $(\Delta T)_\sigma$, which are needed to maintain a fixed operating expense, can be estimated, respectively, by

$$(\Delta T)_\Delta = 812.8(2\omega)^*_\Delta,\ °C \tag{32}$$

$$(\Delta T)_\sigma = 812.8(2\omega)^*_\sigma,\ °C \tag{33}$$

The results for maximum separation are shown in Tables 1a, 1b, and 1c, while those for maximum production rate are given in Table 2a, 2b, and 2c.

DISCUSSION AND CONCLUSIONS

Comparison of separations $\Delta_{N,max}$ and Δ_N, obtainable at the best plate-spacing $(2\omega)^*_\Delta$ and at $2\omega = 0.0406$ cm, respectively, under various flow rates σ and feed concentrations C_0 with fixed operating expense ($\Delta T/2\omega = 812.8°C/cm$), are shown in Tables 1a, 1b, and 1c for $N = 10$, 20, and

TABLE 1a

Comparison of Separations, $\Delta_{N,max}$ and Δ_N, Obtained at the Optimum Plate-Spacing $(2\omega)^*_\Delta$ and at $2\omega = 4.06 \times 10^{-2}$ cm, Respectively, with $N = 10$ ($W_\Delta = 1.1435582$)[a]

			$C_0 = 0.1$		$C_0 = 0.3$		$C_0 = 0.5$		$C_0 = 0.7$		$C_0 = 0.9$	
σ (g/h)	$(2\omega)^*_\Delta \times 10^2$ (cm)	$(\Delta T)_\Delta$ (°C)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)
0.1	3.45	28.0	5.99	8.60	11.83	16.99	12.70	18.23	9.86	14.16	3.95	5.68
0.2	3.72	30.2	5.64	6.32	11.15	12.48	11.97	13.40	9.29	10.41	3.73	4.17
0.4	4.02	32.7	4.63	4.64	9.16	9.17	9.83	9.85	7.63	7.65	3.06	3.45
(0.438)	(4.06)	(33.0)	(4.46)	(4.46)	(8.81)	(8.81)	(9.46)	(9.46)	(7.34)	(7.34)	(2.95)	(2.95)
0.8	4.34	35.3	3.21	3.41	6.33	6.74	6.80	7.24	5.28	5.62	2.12	2.25
1.6	4.69	38.1	1.93	2.51	3.82	4.95	4.10	5.32	3.18	4.13	1.28	1.66
3.2	5.06	41.1	1.07	1.84	2.11	3.64	2.27	3.91	1.76	3.03	0.71	1.22
6.4	5.47	44.5	0.56	1.35	1.11	2.78	1.09	2.87	0.89	2.23	0.37	0.89
12.8	5.91	48.0	0.29	0.95	0.57	1.97	0.61	2.11	0.48	1.64	0.19	0.66

[a] W_Δ was calculated from Eq. (20).

TABLE 1b

Comparison of Separations, $\Delta_{N,max}$ and Δ_N, Obtained at the Optimum Plate-Spacing $(2\omega)^*_\Delta$ and at $2\omega = 4.06 \times 10^{-2}$ cm, Respectively, with $N = 20$ ($W_\Delta = 1.0721517$)[a]

			$C_0 = 0.1$		$C_0 = 0.3$		$C_0 = 0.5$		$C_0 = 0.7$		$C_0 = 0.9$	
σ (g/h)	$(2\omega)^*_\Delta \times 10^2$ (cm)	$(\Delta T)_\Delta$ (°C)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)
0.1	3.19	25.9	6.03	11.89	11.92	23.47	12.79	25.19	9.94	19.57	3.98	7.85
0.2	3.45	28.0	6.01	8.73	11.87	17.25	12.74	18.51	9.89	14.38	3.97	5.77
0.4	3.72	30.2	5.71	6.42	11.28	12.68	12.11	13.61	9.40	10.57	3.77	4.24
0.8	4.02	32.7	4.71	4.72	9.30	9.32	9.94	10.00	7.75	7.76	3.11	3.11
(0.875)	(4.06)	(33.0)	(4.53)	(4.53)	(8.95)	(8.95)	(9.60)	(9.60)	(7.46)	(7.46)	(2.99)	(2.99)
1.6	4.34	35.3	3.25	3.47	6.41	6.85	6.88	7.35	5.35	5.71	2.14	2.29
3.2	4.69	38.1	1.95	2.55	3.85	5.03	4.13	5.40	3.21	4.19	1.29	1.68
6.4	5.06	41.1	1.07	1.87	2.12	3.70	2.28	3.97	1.77	3.08	0.71	1.24
12.8	5.47	44.5	0.56	1.38	1.11	2.72	1.32	2.92	0.93	2.26	0.37	0.91

[a] W_Δ was calculated from Eq. (20).

TABLE 1c

Comparison of Separations, $\Delta_{N,max}$ and Δ_N, obtained at the Optimum Plate-Spacing $(2\omega)^*_\Delta$ and at $2\omega = 4.06 \times 10^{-2}$ cm, Respectively, with $N = 40$ ($W_\Delta = 1.0361618$)[a]

			$C = 0.1$		$C = 0.3$		$C = 0.5$		$C = 0.7$		$C = 0.9$	
σ (g/h)	$(2\omega)^*_\Delta \times 10^2$ (cm)	$(\Delta T)_\Delta$ (°C)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)	Δ_N (%)	$\Delta_{N,max}$ (%)
0.1	2.96	24.1	6.04	16.30	11.92	32.20	12.79	34.56	9.94	26.84	3.98	10.76
0.2	3.19	25.9	6.04	11.98	11.92	23.66	12.79	25.40	9.94	19.72	3.98	7.91
0.4	3.45	28.0	6.02	8.80	11.89	17.39	12.76	18.66	9.91	14.49	3.97	5.81
0.8	3.72	30.2	5.74	6.47	11.34	12.78	12.18	13.71	9.46	10.65	3.79	4.27
1.6	4.02	32.7	4.75	4.75	9.38	9.39	10.06	10.08	7.82	7.83	3.13	3.14
(1.74)	(4.06)	(33.0)	(4.58)	(4.58)	(9.05)	(9.05)	(9.71)	(9.71)	(7.54)	(7.54)	(3.02)	(3.02)
3.2	4.34	35.3	3.27	3.49	6.45	6.90	6.93	7.41	5.38	5.75	2.16	2.31
6.4	4.69	38.1	1.96	2.57	3.86	5.07	4.15	5.44	3.22	4.23	1.29	1.69
12.8	5.07	41.2	1.08	1.89	2.13	3.73	2.28	4.00	1.77	3.11	0.71	1.25

[a] W_Δ was calculated from Eq. (20).

40, respectively. It is seen from these tables that the best plate-spacing for maximum separation increases as the flow rate increases. The temperature differences which should be maintained to keep the operating expense fixed corresponding to $(2\omega)^*_\Delta$ are also delineated in these tables. The improvement in separation is really obtained, especially for $(2\omega)^*_\Delta$, far from 0.0406 cm, which is the plate spacing employed in Yeh and Yang's work (9). When $N = 10$ and $\sigma = 0.438$ g/h, $(2\omega)^*_\Delta = 2\omega = 0.0406$ cm and $\Delta_{N,max} = \Delta_N$ (see Table 1a). Therefore, Yeh and Yang's experimental system operating with this column number and at this flow rate is exactly the case where the plate spacing is optimum and the separation is maximum. Similarly, when $N = 20$ and $\sigma = 0.875$ g/h, $(2\omega)^*_\Delta = 2\omega = 0.0406$ cm and $\Delta_{N,max} = \Delta_N$ (see Table 1b); also when $N = 40$ and $\sigma = 1.74$ g/h, $(2\omega)^*_\Delta = 2\omega = 0.0406$ cm and $\Delta_{N,max} = \Delta_N$ (see Table 1c). It is also noted from these tables that increasing column number N of a Frazier scheme, in addition to increasing the degrees of separations Δ_N and $\Delta_{N,max}$, as well as decreasing the optimum plate-spacing $(2\omega)^*_\Delta$ and its corresponding temperature difference $(\Delta T)_\Delta$, also increases the improvement in separation. For examples: when $N = 10$, $\sigma = 0.1$ g/h and $C_0 = 0.5$, $\Delta_N = 12.70\%$ and $\Delta_{N,max} = 18.23\%$, thus, $(\Delta_{N,max} - \Delta_N)/\Delta_N = 43.5\%$; while as $N = 40$, $\sigma = 0.1$ g/h and $C_0 = 0.5$, $\Delta_N = 12.79\%$ and $\Delta_{N,max} = 34.56\%$, thus, $(\Delta_{N,max} - \Delta_N)/\Delta_N = 170\%$.

Comparison of production rates σ_{max} and σ, obtainable at the best plate-spacing $(2\omega)^*_\sigma$ and at $2\omega = 0.0406$ cm, respectively, under various feed concentrations and degrees of separation with $\Delta T/2\omega = 812.8$°C/cm, are given in Tables 2a, 2b, and 2c for $N = 10$, 20, and 40, respectively. It is shown in these tables that the best plate-spacing $(2\omega)^*_\sigma$ for maximum production rate increases when the specified degree of separation decreases. The temperature differences which should be maintained to keep the oper-

TABLE 2a
Comparison of Production Rates σ_{max} and σ, Obtained at $(2\omega)^*_\sigma$ and at $2\omega = 4.06 \times 10^{-2}$ cm, Respectively, with $N = 10$ ($W_\sigma = 0.26145$)[a]

	Δ_N (%)								
Δ_N/A	$C_0 = 0.1$	$C_0 = 0.3$	$C_0 = 0.5$	$C_0 = 0.7$	$C_0 = 0.9$	σ (g/h)	$(2\omega)^*_\sigma \times 10^2$ (cm)	σ_{max} (g/h)	$(\Delta T)_\sigma$ (°C)
1	0.36	0.71	0.76	0.59	0.24	10.24	7.62	126.97	61.9
2	0.72	1.42	1.52	1.18	0.47	4.94	6.41	26.69	52.1
4	1.44	2.84	3.04	2.36	0.95	2.29	5.39	5.61	45.6
8	2.87	5.67	6.09	4.73	1.90	0.94	4.53	1.18	36.8
(12.42)	(4.46)	(8.81)	(9.46)	(7.34)	(2.95)	(0.438)	(4.06)	(0.438)	(33.0)
16	5.74	11.34	12.18	9.46	3.79	0.18	3.81	0.248	31.0

[a] W_σ was calculated from Eq. (25).

TABLE 2b
Comparison of Production Rates σ_{max} and σ, Obtained at $(2\omega)^*_\sigma$ and at $2\omega = 4.06 \times 10^{-2}$ cm, Respectively, with $N = 20$ ($W_\sigma = 0.24825$)[a]

Δ_N/A	Δ_N (%) $C_0 = 0.1$	$C_0 = 0.3$	$C_0 = 0.5$	$C_0 = 0.7$	$C_0 = 0.9$	σ (g/h)	$(2\omega)^*_\sigma \times 10^2$ (cm)	σ_{max} (g/h)	$(\Delta T)_\sigma$ (°C)
1	0.36	0.71	0.76	0.59	0.24	20.51	7.66	262.92	62.3
2	0.72	1.42	1.52	1.18	0.47	9.92	6.44	55.27	52.3
4	1.44	2.84	3.04	2.36	0.95	4.61	5.41	11.62	44.0
8	2.87	5.67	6.09	4.73	1.90	1.92	4.55	2.44	37.0
(12.63)	(4.53)	(8.95)	(9.60)	(7.46)	(2.99)	(0.875)	(4.06)	(0.875)	(33.0)
16	5.74	11.34	12.18	9.46	3.79	0.39	3.83	0.51	31.0

[a] W_σ was calculated from Eq. (25).

ating expense fixed corresponding to $(2\omega)^*_\sigma$ are also delineated in these tables. The improvement in production rate is really obtained, especially for $(2\omega)^*_\sigma$ far from 0.0406 cm. When $N = 10$ and $\Delta_N/A = 12.42$ (Table 2a), or when $N = 20$ and $\Delta_N/A = 12.63$ (Table 2b), or when $N = 40$ and $\Delta_N/A = 12.76$ (Table 2c), $(2\omega)^*_\sigma = 2\omega = 0.0406$ cm and $\sigma_{max} = \sigma$ (0.438 g/h for $N = 10$; 0.875 g/h for $N = 20$; 1.74 g/h for $N = 40$). Thus, Yeh and Yang's experimental system operating at these specified feed concentrations and degrees of separations is exactly the case in which the plate spacing is optimum and the product rate is maximum. It is also noted from Tables 2a, 2b, and 2c that increasing the column number N of a Frazier scheme greatly increases the production rates σ_{max} and σ but only barely increases the improvement in production rate. For example: When $N = 10$ and $\Delta_N/A = 1$, $\sigma = 10.24$ g/h and $\sigma_{max} = 126.97$ g/h, thus, $(\sigma_{max} - \sigma)/\sigma = 11.4$; while as $N = 40$ and $\Delta_N/A = 1$, $\sigma = 41.06$ g/h and σ_{max}

TABLE 2c
Comparison of Production Rates σ_{max} and σ, Obtained at $(2\omega)^*_\sigma$ and at $2\omega = 4.06 \times 10^{-2}$ cm, Respectively, with $N = 40$ ($W_\sigma = 0.24150$)[a]

Δ_N/A	Δ_N (%) $C_0 = 0.1$	$C_0 = 0.3$	$C_0 = 0.5$	$C_0 = 0.7$	$C_0 = 0.9$	σ (g/h)	$(2\omega)^*_\sigma \times 10^2$ (cm)	σ_{max} (g/h)	$(\Delta T)_\sigma$ (°C)
1	0.36	0.71	0.76	0.59	0.24	41.06	7.67	535.25	62.3
2	0.72	1.42	1.52	1.18	0.47	19.86	6.45	112.52	52.4
4	1.44	2.84	3.04	2.36	0.95	9.24	5.42	23.65	44.1
8	2.87	5.67	6.09	4.73	1.90	3.87	4.56	4.97	37.1
(12.76)	(4.58)	(9.05)	(9.71)	(7.54)	(3.02)	(1.74)	(4.06)	(1.74)	(33.0)
16	5.74	11.34	12.18	9.46	3.79	0.80	3.84	1.05	31.2

[a] W_σ was calculated from Eq. (25).

$= 535.25$ g/h, thus, $(\sigma_{max} - \sigma)/\sigma = 12.04$. Further, the column number minimally affects the optimum plate-spacing $(2\omega)^*_\sigma$ for maximum production rate, as well as its corresponding temperature difference $(\Delta T)_\sigma$.

It was mentioned before that $(2\omega)^*_\Delta$ increases as the flow rate increases or the column number decreases, and that $(2\omega)^*_\sigma$ increases when Δ_N/A decreases, although plate spacing in a thermal diffusion column is generally so small that changing $(2\omega)^*_\Delta$ or $(2\omega)^*_\sigma$ will not cause any additional fixed charge. However, increasing $(2\omega)^*_\Delta$ or $(2\omega)^*_\sigma$ will lead to an increase of $(\Delta T)_\Delta$ or $(\Delta T)_\sigma$ in order to maintain the operating cost, i.e., $\Delta T/2\omega$ constant, therefore some additional cost may be needed to maintain the higher ΔT. Moreover, since the boiling and freezing points of water are 100 and 0°C, respectively, the temperature at the hot plate must be kept lower than 100°C while that at the cold plate should be maintained higher than 0°C, and accordingly, both $(\Delta T)_\Delta$ and $(\Delta T)_\sigma$ are less than 100°C in practical applications.

SYMBOLS

A	a constant defined by Eq. (5) with C_0 and K_{eq} as parameters
a	a constant defined by Eq. (17) (g/h·cm^5)
B	column width (cm)
b	a constant defined by Eq. (18) (g/h·cm^{12})
C	fractional mass concentration of heavy water (D_2O)
C_i	C in the ith column of a Frazier scheme
C_0	C in the feed streams
$C_{B,i}$	C in the bottom-product stream from the ith column
$C_{T,i}$	C in the top-product stream from the ith column
D	ordinary diffusion coefficient (cm^2/s)
g	gravitational acceleration (cm/s^2)
H	transport coefficient defined by Eq. (2) (g/s)
$J_{x\text{-OD}}$	mass flux of D_2O in x-direction due to ordinary diffusion (g/cm^2·s)
$J_{x\text{-TD}}$	mass flux of D_2O in x-direction due to thermal diffusion (g/cm^2·s)
$J_{z\text{-OD}}$	mass flux of D_2O in z-direction due to ordinary diffusion (g/cm^2·s)
K	transport coefficient defined by Eq. (3) (g·cm/s)
K_{eq}	mass-fractional equilibrium constant of H_2O–HDO–D_2O system
L	column length (cm)
N	column number of a Frazier scheme
T_m	mean absolute temperature (K)
ΔT	difference in temperature of hot and cold plates (K)
$(\Delta T)_\Delta$	ΔT for maximum separation (K)
$(\Delta T)_\sigma$	ΔT for maximum production rate (K)
W_Δ	value determined from Eq. (21)

W_σ value determined from Eq. (26)
z axis of transport direction (cm)

Greek Letters

α thermal diffusion constant
β $-(\partial\rho/\partial T)$ evaluated at T_m under constant pressure ($g/cm^3 \cdot K$)
Δ_i $C_{B,i} - C_{T,i}$
$\Delta_{N,max}$ maximum value of Δ_N
μ absolute viscosity (g/cm·s)
ρ mass density evaluated at T_m (g/cm^3)
σ mass flow rate (g/s)
σ_{max} maximum value of σ (g/s)
(2ω) plate spacing, distance between hot and cold plates (cm)
$(2\omega)^*_\Delta$ optimum (2ω) for maximum separation (cm)
$(2\omega)^*_\sigma$ optimum (2ω) for maximum production rate (cm)
τ_i transport of D_2O along z-direction in the ith column (g/s)

REFERENCES

1. W. P. Bebbington and V. R. Thayer, "Production of Heavy Water," *Chem. Eng. Prog.*, *55*(9), 70 (1959).
2. K. Clusius and G. Dickel, "New Process for Separation of Gas Mixtures and Isotopes," *Naturwissenschaften*, *26*, 546 (1938).
3. K. Clusius and G. Dickel, "The separation-Tube Process for Liquid," *Ibid.*, *27*, 148 (1939).
4. Frazier, "Analysis of Transverse-Flow Thermal Diffusion," *Ind. Eng. Chem., Process Des. Dev.*, *1*, 237 (1962).
5. W. H. Furry, R. C. Jones, and L. Onsager, "On the Theory of Isotope Separation by Thermal Diffusion," *Phys. Rev.*, *55*, 1083 (1939).
6. G. M. Murphy (Ed.), *Production of Heavy Water* (National Nuclear Energy Series, III-4F), McGraw-Hill, New York, NY, 1955.
7. G. D. Rabinovich, "Theory of Thermodiffusion Separation According to the Frazier Scheme," *Inzh.-Fiz. Zh.*, *31*, 31 (1976).
8. A. V. Sovorov and G. D. Rabinovich, "Theory of a Thermal Diffusion Apparatus with Transverse Flows," *Ibid.*, *41*, 231 (1981).
9. H. M. Yeh and S. C. Yang, "The Enrichment of Heavy Water in a Batch-Type Thermal Diffusion Column," *Chem. Eng. Sci.*, *39*, 1277 (1984).
10. H. M. Yeh and S. C. Yang, "The Enrichment of Heavy Water in a Continuous-Type Inclined Thermal Diffusion Column," *Sep. Sci. Technol.*, *20*, 101 (1985).
11. H. M. Yeh and S. C. Yang, "Experimental Studies on the Separation of Deuterium Oxide in Continuous Thermal Diffusion Column for Low Concentration Range," *Ibid.*, *20*, 687 (1985).
12. H. M. Yeh, S. C. Yang, and S. W. Tsai, "The Simplified Equation for the Enrichment of Heavy Water in a Batch-Type Thermal Diffusion Column," *Ibid.*, *22*, 1463 (1987).
13. H. M. Yeh and S. M. Chang, "Separation of Heavy Water by Vapor-Phase Thermal Diffusion Coupled with Distillation and Condensation," *Ibid.*, *29*, 1769 (1994).

14. H. M. Yeh, "The Optimum Plate-Spacing for the Best Performance in Flat-Plate Thermal Diffusion Columns," *Chem. Eng. Sci.*, *49*, 2027 (1994).
15. H. M. Yeh, "Enrichment of Heavy Water in Flat-Plate Thermal Diffusion Columns of the Frazier Scheme Inclined for Improved Performance," *Sep. Sci. Technol.*, *30*, 1025 (1995).
16. H. M. Yeh, "The Optimum Design of Inclined Frazier Scheme for Enrichment of Heavy Water by Thermal Diffusion," *Ibid.*, submitted.

Received by editor December 13, 1994
Revision received January 31, 1996